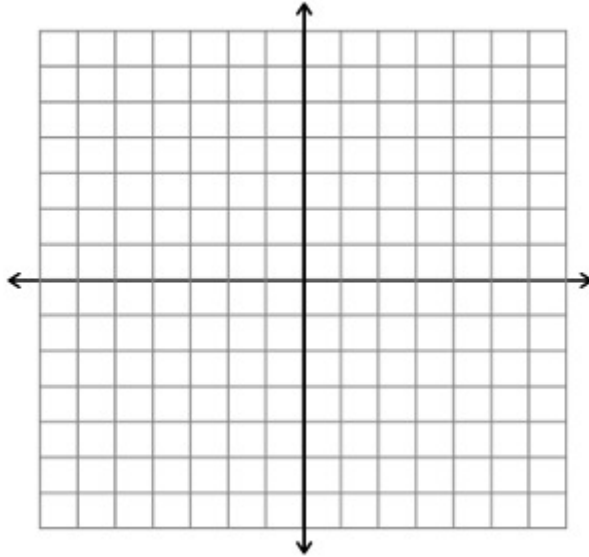


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**In  $\triangle ABC$ ,  $A(1, 6)$ ,  $B(5, 4)$ ,  $C(-3, -2)$  are the vertices of the triangle.**

1. Sketch the triangle on the given coordinate plane below. (3 pts)



2. Find equations of  $\overline{AB}$  in standard form, slope-y intercept form, and slope - point form. (15 pts)

Slope of  $AB \rightarrow m = \frac{4-6}{5-1} = -\frac{2}{4} = -\frac{1}{2}$

Slope-point form:  $y - 6 = -\frac{1}{2}(x - 1)$

Slope y-intercept form:  $y = -\frac{1}{2}x + b$  by using  $(1, 6)$ , we get

$$6 = -\frac{1}{2}(1) + b \rightarrow b = \frac{13}{2}$$

So, we get  $y = -\frac{1}{2}x + \frac{13}{2}$

Standard form:  $Ax + By + C = 0$

From the slope y-int form,  $\frac{1}{2}x + y - \frac{13}{2} = 0 \rightarrow x + 2y - 13 = 0$

3. Find an equation of perpendicular bisector of  $\overline{AB}$ . (10 pts)

We need the midpoint of AB and slope of AB.

Since the slope of AB was  $-\frac{1}{2}$  from the previous question, slope of the line perpendicular is 2.

$$\text{Midpoint} = \left( \frac{1+5}{2}, \frac{6+4}{2} \right) = (3, 5)$$

Eq.

$$y - 5 = 2(x - 3)$$

4. Find an equation of perpendicular bisector of  $\overline{BC}$ . (10 pts)

We need the midpoint of BC and slope of BC.

$$m_{BC} = \frac{-2 - 4}{-3 - 5} = \frac{-6}{-8} = \frac{3}{4}$$

Since the slope of AB was  $\frac{3}{4}$  from the previous question, slope of the line perpendicular is  $\frac{4}{3}$ .

$$\text{Midpoint} = \left( \frac{-3+5}{2}, \frac{-2+4}{2} \right) = (1, 1)$$

Eq.

$$y - 1 = \frac{3}{4}(x - 1)$$

5. Find the coordinates of intersection of perpendicular bisectors of  $\overline{AB}$  and  $\overline{BC}$ . (10 pts)

By setting y's equal to each other,

$$2(x - 3) + 5 = \frac{3}{4}(x - 1) + 1$$

$$\begin{aligned} 2x - 6 + 5 &= \frac{3x}{4} - \frac{3}{4} + 1 \\ \frac{5}{4}x &= \frac{5}{4} \\ x &= 1 \end{aligned}$$

Then, by putting x=1 for any equation, you will get y=1 as well.

(1, 1)

6. Find the distance between the intersection from #5 and B. (8 pts)

$$d = \sqrt{(5-1)^2 + (4-1)^2} = \sqrt{4^2 + 3^2} = 5$$

7. Find an equation of altitude from A. (10 pts)

Altitude  $\rightarrow$  perpendicular to base

Since the slope of BC was  $\frac{3}{4}$  from #4, slope of altitude is  $-\frac{4}{3}$ .  
So,

$$y - 6 = -\frac{4}{3}(x - 1)$$

using point A(1, 6)

8. Find an equation of altitude from B. (10 pts)

Altitude  $\rightarrow$  perpendicular to base

Since the slope of AB was  $-\frac{1}{2}$  from #2, slope of altitude is 2.  
So,

$$y - 4 = 2(x - 5)$$

using point B(5, 4)

9. Let M be the mid point of  $\overline{AB}$ . If  $A(2, a)$  and  $M(a-1, 5)$ , find the coordinates of B. (9 pts)

Since from A to M, you added  $(a-3, 5-a)$ , you need to add another  $(a-3, 5-a)$  to M to get B. So,

The point B becomes

$$(a-1 + (a-3), 5 + (5-a)) = (2a-4, 10-a)$$

10. C is a point on  $\overline{AB}$  and between A and B with  $A(3, 5)$  and  $B(10, -9)$ .  
If  $AC:CB = 2:5$ , find the coordinates of C. (15 pts)